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Order filtrations of the edge algebra

Xiaojing Lyu[1] · **Tao Qian**[2] · **B.-W. Schulze**[3]

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Abstract By edge algebra we understand a pseudo-differential calculus on a manifold with edge. The operators have a two-component principal symbolic hierarchy which determines operators up to lower order terms. Those belong to a filtration of the corresponding operator spaces. We give a new characterisation of this structure, based on an alternative representation of edge amplitude functions only containing holomorphic edge-degenerate Mellin symbols.

Contents

✉ B.-W. Schulze
schulze@math.uni-potsdam.de

Xiaojing Lyu
xiaojinglv2003@tute.edu.cn

Tao Qian
tqian1958@gmail.com

1 College of Science, Tianjin University of Technology and Education, Tianjin 300222, People's Republic of China

2 Faculty of Science and Technology, University of Macau, Taipa, Macau, China SAR

3 Institute of Mathematics, University of Potsdam, 14469 Potsdam, Germany

Birkhäuser

1 Introduction

The calculus of operators on a manifold with edge has been introduced in [27], including its principal symbolic hierarchy and trace and potential operators. Those determine ellipticity and parametrices within the calculus. The approach has been inspired by pseudo-differential boundary value problems in the sense of Vishik and Eskin [34,35], Eskin [7, Subsection15], Boutet de Monvel [1], and also by Kondratyev [16], Rabinovich [19], on operators on manifolds with conical singularities. The subsequent development lead to deeper insight and numerous generalisations, see, for instance, Rempel and Schulze [20], the monograph [26], moreover, Dorschfeldt [5], Schrohe and Schulze [31], Gil et al. [10], Schulze [25], Coriasco and Schulze [4], Chang et al. [2,3], Rungrottheera [21,22]. The edge calculus is motivated by many interesting applications, see the monographs of Kapanadze and Schulze [15], Harutyunyan and Schulze [11], the article Flad and Harutyunyan [8], or Chang et al. [3].

We study edge symbols in terms of holomorphic Mellin symbols, cf. [10], and characterise the filtration of edge operator spaces with respect to orders in a new way. Our approach is motived by expected analogous structures for higher corner pseudo-differential operators where Mellin representations on singular cones up to exits to infinity seem to be most natural. The present paper is organised as follows.

In Sect. 2 we first recall some notation on weighted distribution spaces in terms of the Mellin transform. Concerning basics we refer to Jeanquartier [14] or [23]. Compared with Mellin quantisations in [24] the main new aspect is that the present definition of operator-valued edge amplitude functions in Definition 2.2 (iii) refers to holomorphic families (2.16) but not on the Mellin quantisation of edge-degenerate operator families (2.6) as is done earlier in studying the edge calculus, cf., for instance, [24]. This has many consequences for managing edge operators, though we do not discuss here all technical changes connected with this modification. The cut-off functions $\omega,\ \omega'$ in this description (2.15) are fixed. Possible changes only contribute operators far from the edge, see Remark 2.3.

Section 3 gives a brief description of spaces of edge operators $L^{\mu}(M, \boldsymbol{g})$ on a manifold M with edge Y and weight data $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$. The meaning is completely analogous to other expositions on the edge calculus, however, as announced before, based on the Mellin version of local edge amplitude functions. We formulate the principal symbolic structure $\sigma = (\sigma_0, \sigma_1)$, consisting of the homogeneous principal edge-degenerate interior symbol σ_0 and the (twisted homogeneous) edge symbol σ_1. The latter relies on the representation from the article [9]. Concerning the nature of σ_1 on the open stretched model cone $(r, x) \in X^{\wedge}$ of local wedges we will return to more details in [18]. The remarkable aspect is that the Mellin representation of σ_1 for $r \to \infty$ concerns Fourier based Sobolev spaces, not Mellin ones, which is just the reason for the present new description of the order filtration of the edge calculus.

In Sect. 4 we establish this filtration, based on the indicated shape of edge amplitude functions. Similarly as in boundary value problems which are known to be a special case of the edge calculus, the aspect of twisted homogeneity make the local symbolic information spread out to the infinite stretched cone $X^{\wedge}$. In elliptic operators this effect is responsible for the nature of elliptic edge conditions, either with Shapiro-Lopatinskij ellipticity, or ellipticity with global projection conditions, cf. [24] or [28]. The main results of Sect. 4 are Theorems 4.1 and 4.2. This allows us to state the filtration of the spaces of edge operators. In Theorem 4.7 we see that the space $L^{-\infty}(M, \boldsymbol{g})$ of edge operators of order $-\infty$ admits different equivalent characterisations, namely, (4.21) and (4.22). In Sect. 5 we recall some necessary material on the cone algebra, in particular, Kegel spaces with and without asymptotics.

2 Edge symbols

Let M be a manifold with edge Y. In particular, Y is a smooth manifold of dimension $q > 0$ such that $M \backslash Y$ is smooth as well, and M is locally near Y described by a Cartesian product

$$X^{\Delta} \times \Omega, \;\; X^{\Delta} := \left(\overline{\mathbb{R}}_{+} \times X\right)/(\{0\} \times X), \tag{2.1}$$

for an open set $\Omega \subseteq \mathbb{R}^q$, corresponding to a chart on Y and a smooth manifold X (closed in our case).

The main ingredient of edge symbols in our calculus are operator functions of the form

$$h(r, y, w, \eta) = \tilde{h}(r, y, w, r\eta) \tag{2.2}$$

for

$$\tilde{h}(r, y, w, \tilde{\eta}) \in C^{\infty}\left(\overline{\mathbb{R}}_{+} \times \Omega, M^{\mu}_{\mathcal{O}}\left(X; \mathbb{R}^q_{\tilde{\eta}}\right)\right), \tag{2.3}$$

with $\mu \in \mathbb{R}$ being the order. The meaning of $M^{\mu}_{\mathcal{O}}(\cdot)$ in (2.3) is as follows. Let $L^{\mu}_{\mathrm{cl}}(X; \mathbb{R}^l_{\lambda})$ be the space of classical parameter-dependent pseudo-differential operators over X of order μ in its natural Fréchet topology. Let $\mathcal{A}(U, E)$ for a Fréchet space E and $U \subseteq \mathbb{C}$ open denote the space of all holomorphic E-valued functions in U. Then $M^{\mu}_{\mathcal{O}}(X; \mathbb{R}^l_{\lambda})$ is the space of all

$$h(w, \lambda) \in \mathcal{A}\left(\mathbb{C}, L^{\mu}_{\mathrm{cl}}\left(X; \mathbb{R}^l_{\lambda}\right)\right)$$

such that

$$h|_{\Gamma_{\beta} \times \mathbb{R}^l} \in L^{\mu}_{\mathrm{cl}}\left(X; \Gamma_{\beta} \times \mathbb{R}^l\right)$$

for every $\beta \in \mathbb{R}$, uniformly in compact β-intervals, where

$$\Gamma_{\beta} := \{w \in \mathbb{C} : \operatorname{Re} w = \beta\}. \tag{2.4}$$

Here, as soon as we talk about uniformity with respect to β in compact intervals we understand boundedness in $L^{\mu}_{\mathrm{cl}}(X; \mathbb{R} \times \mathbb{R}^l)$ when Γ_{β} is identified with $\mathbb{R}$ via $\Gamma_{\beta} \ni w = \beta + i\rho \to \rho \in \mathbb{R}$. This has the consequence that we can replace "uniformly" by "smooth dependence" in β with values in $L^{\mu}_{\mathrm{cl}}(X; \Gamma_{\beta} \times \mathbb{R}^l)$. More details may be found in Seiler [32].

We systematically employ pseudo-differential operators with operator-valued symbols. Those can be of the form (2.2), or, more generally,

$$f(r, y, w, \eta) \in C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, L^\mu_{\mathrm{cl}}\left(X; \Gamma_\beta \times \mathbb{R}^q\right)\right), \tag{2.5}$$

based on the weighted Mellin transform

$$M_\gamma u(w) = \int_0^\infty r^w u(r) \frac{dr}{r}\Big|_{\Gamma_{\frac{1}{2}-\gamma}}$$

with $\gamma \in \mathbb{R}$ being a given weight. We then write

$$\mathrm{Op}^\gamma_M(f)(y, \eta)u(r) = \int_{\mathbb{R}} \int_0^\infty \left(\frac{r}{r'}\right)^{-(1/2-\gamma+i\rho)} f\left(r, y, 1/2-\gamma+i\rho, \eta\right) u(r') \frac{dr'}{r'} đ\rho,$$

$đ\rho = (2\pi)^{-1} d\rho$, for functions $u(r') \in C_0^\infty(\mathbb{R}_+, C^\infty(X))$. Later on the action is extended to more general distributions in $\mathbb{R}_+$.

Operator families (2.3) appear in the following Mellin quantisation results, cf. [24, Theorem 3.2.7], or [10, Theorem 2.3]. Let

$$p(r, y, \rho, \eta) := \tilde{p}(r, y, r\rho, r\eta), \quad \tilde{p}(r, y, \tilde{\rho}, \tilde{\eta}) \in C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, L^\mu_{\mathrm{cl}}\left(X; \mathbb{R}^{1+q}_{\tilde{\rho},\tilde{\eta}}\right)\right). \tag{2.6}$$

Then there is an $h(r, y, w, \eta)$ like (2.2), (2.3) such that

$$\mathrm{Op}_r(p)(y, \eta) = \mathrm{Op}^\gamma_M(h)(y, \eta) \mod C^\infty\left(\Omega, L^{-\infty}\left(X^\wedge; \mathbb{R}^q_\eta\right)\right) \tag{2.7}$$

for every $\gamma \in \mathbb{R}$. Conversely, for any h we find a p with the indicated properties such that (2.7) holds, and the resulting operator functions $\tilde{p}$ and $\tilde{h}$ are unique modulo $C^\infty(\overline{\mathbb{R}}_+ \times \Omega, L^{-\infty}(X; \mathbb{R}^{1+q}_{\tilde{\rho},\tilde{\eta}}))$ and $C^\infty(\overline{\mathbb{R}}_+ \times \Omega, M^{-\infty}_{\mathcal{O}}(X; \mathbb{R}^q_{\tilde{\eta}}))$, respectively.

Remark 2.1 For purposes below we formulate a simple consequence of the latter Mellin quantisation theorem. For

$$p(r, y, \rho, \eta) := \tilde{p}(r, y, r\rho, r\eta), \quad \tilde{p}(r, y, \tilde{\rho}, \tilde{\eta}) \in C^\infty(\overline{\mathbb{R}}_+ \times \Omega, L^{\mu-j}_{\mathrm{cl}}(X; \mathbb{R}^{1+q}_{\tilde{\rho},\tilde{\eta}}))$$

for any fixed $j \in \mathbb{N}$, *we find an*

$$h(r, y, w, \eta) = \tilde{h}(r, y, w, r\eta) \quad \text{for} \quad \tilde{h}(r, y, w, \tilde{\eta}) \in C^\infty(\overline{\mathbb{R}}_+ \times \Omega, M^{\mu-j}_{\mathcal{O}}(X; \mathbb{R}_{\tilde{\eta}}))$$

such that

$$\mathrm{Op}_r(r^j p)(y, \eta) = \mathrm{Op}^\gamma_M(r^j h)(y, \eta) \mod C^\infty(\Omega, L^{-\infty}(X^\wedge; \mathbb{R}^q_\eta)).$$

Conversely, for h we find a p with the indicated properties.

Recall that there is well-known kernel cut-off operator based on the Mellin transform

$$\mathbb{V}_M(\psi): C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, L^\mu_{\mathrm{cl}}\left(X; \Gamma_\beta \times \mathbb{R}^q\right)\right) \to C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, M^\mu_{\mathcal{O}_w}\left(X; \mathbb{R}^q\right)\right)$$

for a cut-off function $\psi \in C_0^\infty(\mathbb{R}_+)$, $\psi \equiv 1$ in a neighbourhood of $r = 1$, such that

$$\mathbb{V}_M(\psi) f|_{\Gamma_\beta} = f \mod C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, L^\mu_{\mathrm{cl}}\left(X; \Gamma_\beta \times \mathbb{R}^q\right)\right).$$

This shows that the space of symbols (2.3) is "nearly" as rich as (2.5). The choice of $\gamma \in \mathbb{R}$ is arbitrary. For normalising weights we often replace γ by $\gamma - n/2$ for $n = \dim X$. In the edge algebra we interpret

$$r^{-\mu}\mathrm{Op}_M^{\gamma-n/2}(h)(y, \eta)$$

for h as in (2.3) as an operator-valued symbol, i.e., an element of

$$S^\mu\left(\Omega \times \mathbb{R}^q; H, \tilde{H}\right) \tag{2.8}$$

for suitable Hilbert spaces of weighted distributions on $X^\wedge := \mathbb{R}_+ \times X$. In concrete cases we set

$$H = \mathcal{K}^{s,\gamma}(X^\wedge), \;\; \tilde{H} = \mathcal{K}^{s-\mu,\gamma-\mu}(X^\wedge), \tag{2.9}$$

cf. the definition of Kegel spaces $\mathcal{K}^{s,\gamma}(X^\wedge)$ in (2.10) below. Symbol spaces (2.8) make sense for general (separable) Hilbert spaces H, $\tilde{H}$ with group action. Here a Hilbert space H is said to be endowed with a group action $\kappa = \{\kappa_\delta\}_{\delta \in \mathbb{R}_+}$ if κ is a one-parameter group of isomorphisms $\kappa_\delta : H \to H$, such that $\kappa_\delta \kappa_{\delta'} = \kappa_{\delta\delta'}$ for every $\delta, \delta' \in \mathbb{R}_+$, where $u \to \kappa_\delta u$ defines an element of $C(\mathbb{R}_{+,\delta}, H)$ for every $u \in H$.

More generally, we also admit Fréchet spaces

$$E = \varprojlim_{j \in \mathbb{N}} E^j,$$

written as a projective limit of Hilbert spaces with continuous embeddings $E^{j+1} \hookrightarrow E^0$ for all j. Then E is said to be endowed with a group action κ if it is a group action on E^0 and $\kappa|_{E^j}$ is a group action on E_j for every $j \in \mathbb{N}$.

Now if H and $\tilde{H}$ are Hilbert spaces with group action κ and $\tilde{\kappa}$, respectively, then (2.8) is the set of all

$$a(y, \eta) \in C^\infty\left(\Omega \times \mathbb{R}^q, \mathcal{L}\left(H, \tilde{H}\right)\right)$$

such that

$$\left\|\tilde{\kappa}^{-1}_{\langle\eta\rangle}\left\{D_y^\alpha D_\eta^\beta a(y, \eta)\right\}\kappa_{\langle\eta\rangle}\right\|_{\mathcal{L}(H, \tilde{H})} \leq c\langle\eta\rangle^{\mu-|\beta|}$$

for all $(y, \eta) \in K \times \mathbb{R}^q$, $K \Subset \Omega$, and multi-indices $\alpha, \beta \in \mathbb{N}^q$, for constants $c = c(\alpha, \beta, K) > 0$.

Let $S^{(\mu)}(\Omega \times (\mathbb{R}^q \backslash \{0\}); H, \tilde{H})$ be the space of all $a_{(\mu)}(y, \eta) \in C^\infty(\Omega \times (\mathbb{R}^q \backslash \{0\}), \mathcal{L}(H, \tilde{H}))$ such that

$$a_{(\mu)}(y, \delta\eta) = \delta^\mu \tilde{\kappa}_\delta a_{(\mu)}(y, \eta)\kappa_\delta^{-1}$$

for all $\delta \in \mathbb{R}_+$. Moreover $S^\mu_{\mathrm{cl}}(\Omega \times \mathbb{R}^q; H, \tilde{H})$ denotes the set of all $a(y, \eta) \in S^\mu(\Omega \times \mathbb{R}^q; H, \tilde{H})$ such that there are elements $a_{(\mu-j)} \in S^{(\mu-j)}(\Omega \times (\mathbb{R}^q \backslash \{0\}); H, \tilde{H})$, $j \in \mathbb{N}$, where

$$a(y, \eta) - \sum_{j=0}^{N} \chi(\eta) a_{(\mu-j)}(y, \eta) \in S^{\mu-(N+1)}\left(\Omega \times \mathbb{R}^q; H, \tilde{H}\right)$$

for every $N \in \mathbb{N}$; here $\chi(\eta)$ is any excision function in $\mathbb{R}^q$.

The Kegel spaces $\mathcal{K}^{s,\gamma}(X^\wedge)$ over $X^\wedge$ for a closed smooth manifold X and $\gamma \in \mathbb{R}$ are defined as follows. For any cut-off function $\omega(r)$ on the r half-axis (i.e., $\omega \in C_0^\infty(\overline{\mathbb{R}}_+)$ is real-valued and $\omega \equiv 1$ close to $r = 0$) we set

$$\mathcal{K}^{s,\gamma}(X^\wedge) := \left\{\omega u + (1-\omega)v : u \in \mathcal{H}^{s,\gamma}(X^\wedge),\ v \in H^s_{\mathrm{cone}}(X^\wedge)\right\}. \tag{2.10}$$

Here $\mathcal{H}^{s,\gamma}(X^\wedge)$ is the completion of $C_0^\infty(\mathbb{R}_+, C^\infty(X))$ with respect to the norm

$$\|u\|_{\mathcal{H}^{s,\gamma}(X^\wedge)} := \left\{\int_{\Gamma_{\frac{n+1}{2}-\gamma}} \left\| R^s(\operatorname{Im} w)(Mu)(w)\right\|^2_{L^2(X^\wedge)} đw\right\}^{\frac{1}{2}}, \tag{2.11}$$

$đw := (2\pi i)^{-1}dw$, for any parameter-dependent elliptic element $R^s(\lambda) \in L^s_{\mathrm{cl}}(X; \mathbb{R}_\lambda)$ that induces a family of isomorphisms

$$R^s(\lambda) : H^t(X) \to H^{t-s}(X)$$

between standard Soblev spaces, for every $t \in \mathbb{R}$. It is well-known that such smoothness reducing families exist and that (2.11) is independent of the choice of R^s, up to equivalence of norms.

Moreover, $H^s_{\mathrm{cone}}(X^\wedge)$ is the space of all $v \in H^s_{\mathrm{loc}}(\mathbb{R} \times X)|_{\mathbb{R}_+ \times X}$ such that for any coordinate neighbourhood $U \subset X$ and a diffeomorphism $\vartheta : U \to V$ to an open subset V of $S^n = \{\tilde{x} \in \mathbb{R}^{n+1} : |\tilde{x}| = 1\}$ we have $(1-\omega)\varphi \circ \chi^{-1} \in H^s(\mathbb{R}^{n+1})$ for every $\varphi \in C_0^\infty(U)$, a cut-off function ω, and $\chi : \mathbb{R}_+ \times U \to \mathbb{R}^{n+1}$ defined by $\chi(r, x) := r\vartheta(x)$.

Now

$$S^\mu\left(\Omega \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}\left(X^\wedge\right), \mathcal{K}^{s-\mu,\gamma-\mu}\left(X^\wedge\right)\right)$$

refers to the group action

$$(\kappa_\delta u)(r,x) = \delta^{(n+1)/2}u(\delta r, x),\ \ \delta \in \mathbb{R}_+, \tag{2.12}$$

on the space $\mathcal{K}^{s,\gamma}(X^\wedge)$ for every $s, \gamma \in \mathbb{R}$. For any strictly positive smooth function $r \to [r]$ on $\mathbb{R}_+$ such that $[r] = r$ for $r > C$ for some $C > 0$ we set

$$\mathcal{K}^{s,\gamma;e}(X^\wedge) := [r]^{-e}\mathcal{K}^{s,\gamma}(X^\wedge), \quad \mathcal{K}^{\infty,\gamma;\infty}(X^\wedge) := \bigcap_{s,e\in\mathbb{R}} \mathcal{K}^{s,\gamma;e}(X^\wedge), \tag{2.13}$$

for every $s, \gamma, e \in \mathbb{R}$, also endowed with (2.12). Concerning subspaces with asymptotics in the following definition, see Sect. 5 below. More material on such spaces is developed in [17].

Definition 2.2 (i) The space of Green symbols $\mathcal{R}^\mu_{\mathrm{G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for $\mu \in \mathbb{R}$, $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$, $\Theta = (\vartheta, 0]$, is defined as the space of all

$$g(y,\eta) \in \bigcap_{s,e\in\mathbb{R}} S^\mu_{\mathrm{cl}}\left(\Omega \times \mathbb{R}^q; \mathcal{K}^{s,\gamma;e}\left(X^\wedge\right), \mathcal{K}_P^{\infty,\gamma-\mu;\infty}\left(X^\wedge\right)\right),$$

such that

$$g^*(y,\eta) \in \bigcap_{s,e\in\mathbb{R}} S^\mu_{\mathrm{cl}}\left(\Omega \times \mathbb{R}^q; \mathcal{K}^{s,-\gamma+\mu;e}\left(X^\wedge\right), \mathcal{K}_Q^{\infty,-\gamma;\infty}\left(X^\wedge\right)\right),$$

for some g-dependent asymptotic types P and Q, where the pointwise formal adjoint refers to the $\mathcal{K}^{0,0}(X^\wedge)$-scalar product.

(ii) The space of smoothing Mellin plus Green symbols $\mathcal{R}^\mu_{\mathrm{M+G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$ and $\Theta = (-(k+1), 0]$, $k \in \mathbb{N}$, is defined as the space of all $m(y,\eta) + g(y,\eta)$ for $g(y,\eta) \in \mathcal{R}^\mu_{\mathrm{G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ and smoothing Mellin symbols $m(y,\eta)$ of the form

$$m(y,\eta) := r^{-\mu}\omega_\eta \sum_{j=0}^{k} r^j \sum_{|\alpha|\le j} \mathrm{Op}_M^{\gamma_{j\alpha}-n/2}\left(f_{j\alpha}\right)(y)\eta^\alpha \omega'_\eta \tag{2.14}$$

for arbitrary $f_{j\alpha} \in C^\infty(\Omega, M^{-\infty}_{R_{j\alpha}}(X))$, cf. (5.2), with Mellin asymptotic types $R_{j\alpha}$, weights $\gamma_{j\alpha} \in \mathbb{R}$, satisfying

$$\gamma - j \le \gamma_{j\alpha} \le \gamma,\ \Pi_{\mathbb{C}} R_{j\alpha} \cap \Gamma_{\frac{n+1}{2}-\gamma_{j\alpha}} = \emptyset,$$

and cut-off functions ω, ω' on the r half-axis where $\omega_\eta(r) := \omega(r[\eta])$. In the case $\Theta := (-\infty, 0]$ we define the corresponding $\mathcal{R}_{\mathrm{M+G}}$-space as the intersection of those for $k \in \mathbb{N}$.

(iii) The space of edge symbols $\mathcal{R}^\mu(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ is defined as the set of all operator functions $a(y,\eta)$ of the form

$$a(y,\eta) = r^{-\mu}\omega \mathrm{Op}_M^{\gamma-n/2}(h)(y,\eta)\omega' + (m+g)(y,\eta) \tag{2.15}$$

for arbitrary cut-off functions ω, ω' on the r half-axis, for an

$$h(r, y, w, \eta) = \tilde{h}(r, y, w, r\eta), \ \ \tilde{h}(r, y, w, \tilde{\eta}) \in C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, M_{\mathcal{O}}^\mu\left(X; \mathbb{R}^q_{\tilde{\eta}}\right)\right), \tag{2.16}$$

and $(m + g)(y, \eta) \in \mathcal{R}^\mu_{\mathrm{M+G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$, $\Theta = (-(k + 1), 0]$, $k \in \mathbb{N} \cup \{\infty\}$.

Remark 2.3 Changing the cut-off functions $\omega, \ \omega'$ in (2.15) leaves remainders of the form $\varphi \mathrm{Op}_r(p_{\mathrm{int}})(y, \eta)\varphi'$ for $\varphi, \varphi' \in C_0^\infty(\mathbb{R}_+)$, and $p_{\mathrm{int}}(r, y, \rho, \eta) \in C^\infty(\mathbb{R}_+ \times \Omega, L^\mu_{\mathrm{cl}}(X; \mathbb{R}^{1+q}_{\rho,\eta}))$. Such terms could be added in the definition of $a(y, \eta)$; however, by a suitable choice of ω, ω' they can be integrated in the Mellin action. So without loss of generality we employ $a(y, \eta)$ in the form (2.15).

Theorem 2.4 [18, Theorem 4.6] *We have*

$$\mathcal{R}^\mu\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right) \subset S^\mu\left(\Omega \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}\left(X^\wedge\right), \mathcal{K}^{s-\mu,\gamma-\mu}\left(X^\wedge\right)\right)$$

for every $s \in \mathbb{R}$.

Theorem 2.4 refers to Definition 2.2 (iii). In [9] it has been proved that edge amplitude functions in the traditional form based on Mellin-edge quantisations are contained in $\mathcal{R}^\mu(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ in the sense of the present definition. In [18] we gave an independent proof of the fact that this inclusion is surjective.

3 The edge algebra

We now turn to edge operators of order $\mu - j, \ j \in \mathbb{N}$, associated with weight data $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$. The properties of the corresponding filtration are the main issue of this section. By definition our manifold M with edge Y contains a neighbourhood $W \supset Y$ with the structure of a locally trivial X^Δ bundle over Y. That means we have a system of singular charts

$$\chi : V \to X^\Delta \times \Omega \tag{3.1}$$

for neighbourhoods $V \subset M$ of points y on the edge, where

$$\chi|_{V \setminus Y} : V \setminus Y \to X^\wedge \times \Omega \tag{3.2}$$

are diffeomorphisms. If $\tilde{V}$ is another neighbourhood of points $\tilde{y}$ and

$$\chi : \tilde{V} \to X^\Delta \times \tilde{\Omega}$$

the corresponding singular charts, then for $V \cap \tilde{V}$ we have restrictions

$$\chi|_{V \cap \tilde{V}} : V \cap \tilde{V} \to X^\Delta \times D, \quad \tilde{\chi}|_{V \cap \tilde{V}} : V \cap \tilde{V} \to X^\Delta \times \tilde{D}$$

for open subsets $D \subseteq \Omega,\ \tilde{D} \subseteq \tilde{\Omega}$, such that the transition maps

$$X^{\Delta} \times D \to X^{\Delta} \times \tilde{D}$$

are bundle isomorphisms between the corresponding (trivial) X^{Δ}-bundles over D and $\tilde{D}$, respectively. Those are fibrewise, i.e., over every $y \in \Sigma$, quotient maps

$$\left(\overline{\mathbb{R}}_+ \times X\right)/(\{0\} \times X) \to \left(\overline{\mathbb{R}}_+ \times X\right)/(\{0\} \times X)$$

for diffeomorphisms

$$C : \overline{\mathbb{R}}_+ \times X \to \overline{\mathbb{R}}_+ \times X, \tag{3.3}$$

where $\overline{\mathbb{R}}_+ \times X$ is regarded as a manifold with smooth boundary. The following local constructions refer to a fixed chart $G \to \mathbb{R}^q$ on Y, $q = \dim Y > 0$.

For a Hilbert space H with group action κ we have the abstract edge space

$$\mathcal{W}^s(\mathbb{R}^q, H),\ \ s \in \mathbb{R}, \tag{3.4}$$

defined as the completion of $\mathcal{S}(\mathbb{R}^q, H)$ with respect to the norm

$$\left\{ \int \langle\eta\rangle^{2s} \left\| \kappa^{-1}_{\langle\eta\rangle} \hat{u}(\eta) \right\|^2_H đ\eta \right\}^{1/2}, \tag{3.5}$$

with $\hat{u}(\eta)$ being the Fourier transform in $\mathbb{R}^q$. We get an equivalent norm when we replace $\langle\eta\rangle$ by $[\eta]$, where $\eta \to [\eta]$ is any strictly positive function such that $|\eta| = [\eta]$ for $|\eta| \geq$ const.

In the case of a Fréchet space E with group action κ we can form the spaces $\mathcal{W}^s(\mathbb{R}^q, E^j)$ and then set

$$\mathcal{W}^s\left(\mathbb{R}^q, E\right) = \varprojlim_{j \in \mathbb{N}} \mathcal{W}^s\left(\mathbb{R}^q, E^j\right). \tag{3.6}$$

Recall that for $\mathcal{W}^\infty(\mathbb{R}^q, \cdot) := \bigcap_{s \in \mathbb{R}} \mathcal{W}^s(\mathbb{R}^q, \cdot)$ we can forget about the group action κ in the definition, i.e., κ may be replaced by id, i.e., $\kappa_\delta = \mathrm{id}_E$ for all $\delta \in \mathbb{R}_+$.
We employ these constructions to so-called (local) weighted edge spaces

$$\mathcal{W}^s\left(\mathbb{R}^q, \mathcal{K}^{s,\gamma}\left(X^\wedge\right)\right) \tag{3.7}$$

of smoothness $s \in \mathbb{R}$ and subspaces

$$\mathcal{W}^s\left(\mathbb{R}^q, \mathcal{K}^{s,\gamma}_P\left(X^\wedge\right)\right) \tag{3.8}$$

with (here constant discrete) asymptotics of type P, cf. notation in Sect. 5. In other words, we apply (3.4) and (3.6) to $H = \mathcal{K}^{s,\gamma}(X^\wedge)$ and $E = \mathcal{K}^{s,\gamma}_P(X^\wedge)$, respectively. Weighted edge spaces (3.7) and subspaces (3.8) with asymptotics have been studied in [23], see also [24]. The abstract version (3.4) has been introduced in [29]. More functional analytic properties have been studied in [12]. We now formulate global spaces

$$H^{s,\gamma}(M) \quad \text{and} \quad H_P^{s,\gamma}(M) \tag{3.9}$$

on a (compact) manifold M with edge Y. The first space of (3.9) is defined as the set of all $u \in H^s_{\mathrm{loc}}(M\backslash Y)$ such that for any singular chart $\chi : V \to X^\Delta \times \mathbb{R}^q$ and the induced $\chi|_{V\backslash Y} : V\backslash Y \to X^\wedge \times \mathbb{R}^q$ we have

$$u|_{V\backslash Y} = f \circ \chi^{-1}|_{V\backslash Y}$$

for some $f \in \mathcal{W}^s(\mathbb{R}^q, \mathcal{K}^{s,\gamma}(X^\wedge))$. Similarly, the second space of (3.9) is the set of all $u \in H^s_{\mathrm{loc}}(M\backslash Y)$ such that

$$u|_{V\backslash Y} = f \circ \chi^{-1}|_{V\backslash Y}$$

for some $f \in \mathcal{W}^s(\mathbb{R}^q, \mathcal{K}_P^{s,\gamma}(X^\wedge))$. Concerning invariance of this definition under transition maps we impose a mild extra condition on the chosen atlas close to the edge, cf. [24]. A similar definition applies when M is a non-compact manifold with edge. In that case instead of (3.9) we write

$$H^{s,\gamma}_{\mathrm{loc}}(M) \quad \text{and} \quad H^{s,\gamma}_{P,\mathrm{loc}}(M), \tag{3.10}$$

respectively, and we have also the corresponding spaces with subscript "comp". An operator $C : C_0^\infty(M\backslash Y) \to C^\infty(M\backslash Y)$ is smoothing in the edge algebra, i.e., $C \in L^{-\infty}(M, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$, if (say for compact M) the operators C, C^* extend to continuous maps

$$C : H^{s,\gamma}(M) \to H_P^{\infty,\gamma-\mu}(M), \quad C^* : H^{s,-\gamma+\mu}(M) \to H_Q^{\infty,-\gamma}(M) \tag{3.11}$$

for every $s \in \mathbb{R}$, for asymptotic types P, Q, depending on C.
On a compact manifold M with edge Y we choose a system of singular charts $\chi_j : V_j \to X^\Delta \times \mathbb{R}^q$, $j = 1, \ldots, N$, of the kind (3.1), where $G_j := V_j \cap Y$ form an open covering $\{G_1, \ldots, G_N\}$ of Y. Let $\{\varphi_1, \ldots, \varphi_N\}$ be a subordinate partition of unity, and let $\{\varphi'_1, \ldots, \varphi'_N\}$ be a system of functions in $C_0^\infty(G_j)$, $\varphi_j \prec \varphi'_j$ for all j. Moreover, fix cut-off functions $\omega, \omega', \omega''$ on M, i.e., continuous functions on M that are smooth on $M\backslash Y$ and $\equiv 1$ close to Y, and supported by a small neighbourhood of Y, where $\omega'' \prec \omega \prec \omega'$.

Definition 3.1 Let M be a compact manifold with edge Y.

(i) The space $L^\mu(M, \boldsymbol{g})$ of edge operators on M for $\mu \in \mathbb{R}$, $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$, is the set of all $A \in L^\mu_{\mathrm{cl}}(M \backslash Y)$ of the form

$$A = \left\{ \sum_{j=1}^{N} A_j + (1-\omega) A_{\mathrm{int}} \left(1 - \omega''\right) + C : A_{\mathrm{int}} \in L^\mu_{\mathrm{cl}}(M \backslash Y),\ C \in L^{-\infty}(M, \boldsymbol{g}), \right.$$
$$\left. A_j = \omega \varphi_j \left(\mathrm{int}\, \chi_j^{-1}\right)_* \mathrm{Op}_y(a_j) \varphi'_j \omega',\ a_j(y, \eta) \in \mathcal{R}^\mu\left(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g}\right) \right\}, \tag{3.12}$$

where $\mathrm{int}\, \chi_j := \chi_j|_{V_j \backslash Y}$.

(ii) By $L^\mu_{\mathrm{M+G}}(M, \boldsymbol{g})$ $(L^\mu_{\mathrm{G}}(M, \boldsymbol{g}))$ we denote the set of all $A \in L^\mu(M, \boldsymbol{g})$ such that $A_{\mathrm{int}} = 0$ and $a_j \in \mathcal{R}^\mu_{\mathrm{M+G}}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ $(\mathcal{R}^\mu_{\mathrm{G}}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g}))$ for all j.

It is well-known, cf. [24], that $A \in L^\mu(M, \boldsymbol{g})$ induces continuous operators

$$A : H^{s,\gamma}(M) \to H^{s-\mu,\gamma-\mu}(M),\quad H^{s,\gamma}_P(M) \to H^{s-\mu,\gamma-\mu}_Q(M) \tag{3.13}$$

for all $s \in \mathbb{R}$ and asymptotic types P certain resulting Q, depending on P and A. Continuity results (3.13) are based on local continuity of $\mathrm{Op}(a)$ for $a \in \mathcal{R}^\mu_{\mathrm{M+G}}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ in weighted edge Sobolev spaces (3.7) or (3.8). The latter follows from Theorem 2.4 also using that $\mathcal{R}^\mu_{\mathrm{M+G}}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ is contained in $S^\mu(\mathbb{R}^q \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}_P(X^\wedge), \mathcal{K}^{s-\mu,\gamma-\mu}_Q(X^\wedge))$ for asymptotic types P with some resulting Q. Continuity results between abstract edge spaces are proved in [23] and [33], an operator-valued analogue of Hwang's proof [13] of the Calderón-Vaillancourt Theorem.

The assumption of compact M in Definition 3.1 has been made for convenience. A small modification allows us also to admit the paracompact case, using a corresponding locally finite system of charts. Instead of the space in (3.13) we then have to take com/loc-analogues, cf. (3.10). Recall that

$$L^\mu_{\mathrm{G}}(M, \boldsymbol{g}) \subset L^\mu_{\mathrm{M+G}}(M, \boldsymbol{g}) \subset L^{-\infty}(M \backslash Y).$$

An operator $A \in L^\mu(M, \boldsymbol{g}) \subset L^\mu_{\mathrm{cl}}(M \backslash Y)$ has its standard homogeneous principal symbol of order μ

$$\sigma_0(A) \in C^\infty(T^*(M \backslash Y) \backslash 0). \tag{3.14}$$

Moreover, since A is edge-degenerate close to Y, in the splitting of variables $(r, x, y) \in \mathbb{R}_+ \times \Sigma \times \mathbb{R}^q$ and covariables (ρ, ξ, η) the function (3.14) takes the form

$$\sigma_0(A)(r, x, y, \rho, \xi, \eta) = r^{-\mu} \tilde{\sigma}_0(A)(r, x, y, r\rho, \xi, r\eta) \tag{3.15}$$

for a function $\tilde{\sigma}_0(A)(r, x, y, \tilde{\rho}, \xi, \tilde{\eta})$ that is homogeneous in $(\tilde{\rho}, \xi, \tilde{\eta}) \neq 0$ of order μ and smooth up to $r = 0$.

Observe that $\sigma_0(A)$ can be locally close to Y expressed in terms of the operator-valued symbol $a(y, \eta) \in \mathcal{R}^\mu(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$, cf. Definition 3.1 (iii). From (2.7) and the subsequent observation in converse direction we see that the Mellin symbol h in (2.15), (2.16) belongs to $C^\infty(\mathbb{R}_+ \times \Omega, L^\mu_{\mathrm{cl}}(X; \mathbb{R}^q_\eta))$. As such it has a parameter-dependent homogeneous principal symbol

$$p_{(\mu)}(r, x, y, \rho, \xi, \eta),\ (\rho, \xi, \eta) \neq 0.$$

Thus $a(y, \eta)$ itself given by (2.15), has a parameter-dependent homogeneous principal symbol, called $\sigma_0(a)$, which is close to $r = 0$ of the form

$$\sigma_0(a)(r, x, y, \rho, \xi, \eta) := r^{-\mu} p_{(\mu)}(r, x, y, \rho, \xi, \eta) = r^{-\mu} \tilde{p}_{(\mu)}(r, x, y, r\rho, \xi, r\eta) \tag{3.16}$$

for a $\tilde{p}_{(\mu)}(r, x, y, \tilde{\rho}, \xi, \tilde{\eta}) \in S^{(\mu)}(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times (\mathbb{R}^{1+n+q}_{\tilde{\rho},\xi,\tilde{\eta}} \setminus \{0\}))$, where $\Sigma \subset \mathbb{R}^n$ corresponds to a chart on X. Later on, when we talk about lower order symbols we also write

$$\sigma_0^\mu(a) := \sigma_0(a) \quad \text{and} \quad \sigma_0^\mu(A) := \sigma_0(A),$$

respectively. Moreover, the edge amplitude functions $a(y, \eta) \in \mathcal{R}^\mu(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ involved in Definition 3.1 have a (twisted) homogeneous principal symbol, namely,

$$\sigma_1(a)(y, \eta) := r^{-\mu} \mathrm{Op}_M^{\gamma - n/2}(h_0)(y, \eta) + \sigma_1(m + g)(y, \eta), \tag{3.17}$$

$\eta \neq 0$, where

$$h_0(r, y, w, \eta) := \tilde{h}(0, y, w, r\eta), \tag{3.18}$$

cf. (2.16), and $\sigma_1(m + g)(y, \eta)$ is the (twisted) homogeneous principal symbol of $(m + g)(y, \eta)$ as a classical operator-valued symbol, i.e.,

$$(m + g)(y, \eta) \in S^\mu_{\mathrm{cl}}\left(\mathbb{R}^q \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}\left(X^\wedge\right), \mathcal{K}^{s-\mu,\gamma-\mu}\left(X^\wedge\right)\right).$$

Using $\sigma_1(\cdot)(y, \eta)$ on edge amplitude functions we obtain $\sigma_1(A)(y, \eta)$ also for the operators $A \in L^\mu(M, \boldsymbol{g})$ themselves. For the definition we may refer to localised and properly supported representatives of operators, e.g., A_j as in Definition 3.1 (i) and to recover left symbols, similarly as for standard (scalar) pseudo-differential operators. For the resulting $a(y, \eta) \in S^\mu_{\mathrm{cl}}(\mathbb{R}^q \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}(X^\wedge), \mathcal{K}^{s-\mu,\gamma-\mu}(X^\wedge))$ we then have

$$\sigma_1(a)(y, \eta) = \lim_{\delta \to \infty} \delta^{-\mu} \kappa_\delta^{-1} a(y, \delta\eta) \kappa_\delta.$$

Summing up the local symbols which contain contributions of a partition of unity on Y we obtain the invariantly defined principal edge symbol $\sigma_1(A)(y, \eta)$, namely,

$$\sigma_1(A)(y, \eta) := \sum_{j=1}^{N} \varphi_j(y) \sigma_1(a_j)(y, \eta), \tag{3.19}$$

see (3.12).

4 The filtration of edge operator spaces

Order filtrations in the edge calculus are well-known and useful for dealing with lower order terms as soon as principal symbols vanish. We realise here the filtration by using an alternative representation of the edge calculus, cf. [9], based on amplitude functions as in Definition 2.2 (ii). At the same time we deepen the insight of the exit symbolic properties of operator-valued Mellin symbols on the infinite cone for $r \to \infty$. We also look at smoothing Mellin plus Green symbols; however, those are standard. In fact, when we define

$$\mathcal{R}^{\mu-j}_{\mathrm{M+G}}\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right) \quad \text{for} \quad \boldsymbol{g} = (\gamma, \gamma - \mu, (-(k+1), 0]), \tag{4.1}$$

for $j \in \mathbb{N} \backslash \{0\}$, we simply ask the homogeneous components of $(m+g)(y, \eta)$ of order l to be vanishing for all $0 \le l \le j-1$. More precisely, we have

$$\mathcal{R}^{\mu}_{\mathrm{M+G}}\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right) \subset S^{\mu}_{\mathrm{cl}}\left(\Omega \times \mathbb{R}^q; \mathcal{K}^{s,\gamma}\left(X^{\wedge}\right), \mathcal{K}^{\infty,\gamma-\mu}\left(X^{\wedge}\right)\right),$$

$s \in \mathbb{R}$, and any $(m+g)(y, \eta)$ has a sequence of homogeneous components

$$\sigma_1^{\mu-j}(m+g)(y, \eta) := (m+g)_{(\mu-j)}(y, \eta), \tag{4.2}$$

where

$$(m+g)_{(\mu-j)}(y, \eta) \in S^{(\mu-j)}\left(\Omega \times \left(\mathbb{R}^q \backslash \{0\}\right); \mathcal{K}^{s,\gamma}\left(X^{\wedge}\right), \mathcal{K}^{\infty,\gamma-\mu}\left(X^{\wedge}\right)\right), \quad j \in \mathbb{N},$$

cf. the generalities on classical operator-valued symbols in Sect. 2, where by notation $\sigma_1^{\mu}(m+g) := \sigma_1(m+g)$. Then the operator family $(m+g)(y, \eta)$ belongs to (4.1) if $\sigma_1^{\mu-l}(m+g)$ vanishes for all $0 \le l \le j-1$. As is well-known, we have

$$\mathcal{R}^{\mu-j}_{\mathrm{M+G}}\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right) = \mathcal{R}^{\mu-j}_{G}\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right)$$

when $j > k$ where k is involved in the weight interval contained in $\boldsymbol{g}$. The weight data $\boldsymbol{g}$ are independent of j. For general $a(y, \eta) \in \mathcal{R}^{\mu}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ the dominating term is the non-smoothing summand

$$r^{-\mu} \omega \mathrm{Op}_M^{\gamma-n/2}(h)(y, \eta)\omega'. \tag{4.3}$$

For a simple model situation in [18] we illustrated the (unexpected) problem of understanding the homogeneous principal symbol of order μ of (4.3) for $r \to \infty$ in the frame of the exit pseudo-differential calculus on $X^{\wedge}$, for $\eta \neq 0$.

Let $\mathcal{R}^{\mu-j}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$, $j \in \mathbb{N}$, for $\boldsymbol{g}$ as in (4.1), be the space of all operator families of the form

$$r^{-\mu} \omega \mathrm{Op}_M^{\gamma-n/2}\left(r^j h\right)(y, \eta)\omega' + (m+g)(y, \eta)$$

for

$$h(r,y,w,\eta)=\tilde{h}(r,y,w,r\eta),\ \ \tilde{h}(r,y,w,\tilde{\eta})\in C^{\infty}(\overline{\mathbb{R}}_{+}\times\Omega,M_{\mathcal{O}}^{\mu-j}(X;\mathbb{R}^{q}_{\tilde{\eta}})),\quad(4.4)$$

and $(m+g)(y,\eta)\in\mathcal{R}^{\mu-j}_{\mathrm{M+G}}(\Omega\times\mathbb{R}^{q},\boldsymbol{g})$. Similarly as in Remark 2.3 we could add a term $\varphi\mathrm{Op}_{r}(p_{\mathrm{int}})(y,\eta)\varphi'$ for $p_{\mathrm{int}}(r,y,\rho,\eta)\in C^{\infty}(\mathbb{R}_{+}\times\Omega,L^{\mu-j}_{\mathrm{cl}}(X;\mathbb{R}^{1+q}_{\rho,\eta}))$ which is contributed when we change $\omega,\ \omega'$.

Theorem 4.1 *Let $a(y,\eta)\in\mathcal{R}^{\mu}(\Omega\times\mathbb{R}^{q},\boldsymbol{g})$ for $\boldsymbol{g}=(\gamma,\gamma-\mu,(-(k+1),0])$, and assume $\sigma_{i}(a)=0,\ \ i=0,1$. Then $a(y,\eta)$ is of the form*

$$a(y,\eta)=r^{-\mu+1}\omega\mathrm{Op}_{M}^{\gamma-n/2}(h_{1})(y,\eta)\omega'+(m_{1}+g_{1})(y,\eta)$$

for

$$h_{1}(r,y,w,\eta)=\tilde{h}_{1}(r,y,w,r\eta),\ \tilde{h}_{1}(r,y,w,\tilde{\eta})\in C^{\infty}\left(\overline{\mathbb{R}}_{+}\times\Omega,M_{\mathcal{O}}^{\mu-1}\left(X;\mathbb{R}^{q}_{\tilde{\eta}}\right)\right)$$

and $(m_{1}+g_{1})(y,\eta)\in\mathcal{R}^{\mu-1}_{\mathrm{M+G}}(\Omega\times\mathbb{R}^{q},\boldsymbol{g})$.

Proof For convenience we consider the case of y-independent symbols. The modifications for the general case are evident. Let us first note that $a(\eta)$ is a family of operators taking values in the space $L^{\mu}(X^{\Delta},\boldsymbol{g})$ for every fixed $\eta\in\mathbb{R}^{q}$, cf. Definition 5.3 (iii) below. Moreover, we have

$$\sigma_{1}(a)(\eta)\in L^{\mu}(X^{\Delta},\boldsymbol{g})_{\mathrm{exit}}\tag{4.5}$$

for every fixed $\eta\in\mathbb{R}^{q}\backslash\{0\}$, cf. Definition 5.3 (iv) below. In fact, $(m+g)(\eta)$ in (2.15) for fixed $\eta\in\mathbb{R}^{q}$ belongs to $L_{\mathrm{M+G}}(X^{\Delta},\boldsymbol{g})$ which is obvious. It remains to observe

$$r^{-\mu}\omega\mathrm{Op}_{M}^{\gamma-n/2}(h)(\eta)\omega'\in L^{\mu}(X^{\Delta},\boldsymbol{g}),\tag{4.6}$$

cf. the expression (5.9) below. In fact, we have

$$h(r,w,\eta)=\tilde{h}(r,w,r\eta)\tag{4.7}$$

for $\tilde{h}(r,w,\tilde{\eta})\in C^{\infty}(\overline{\mathbb{R}}_{+},M^{\mu}_{\mathcal{O}_{w}}(X;\mathbb{R}^{q}_{\tilde{\eta}}))$. Thus, for fixed η the function (4.7) belongs to $C^{\infty}(\overline{\mathbb{R}}_{+},M^{\mu}_{\mathcal{O}_{w}}(X))$ and hence (4.6) just corresponds to the first summand on the right of (5.9). Setting $p=0$, we see that (4.6) holds. In order to show (4.5) we first note that $\sigma_{1}(m+g)(\eta)\in L_{\mathrm{M+G}}(X^{\Delta},\boldsymbol{g})$. Moreover, the technique of the proof of [18, Lemma53] shows that for any fixed $\eta\neq0$ the operator $r^{-\mu}\mathrm{Op}_{M}^{\gamma-n/2}(h_{0})(\eta)$ is of the form (5.11). From [6, Subsection 3.5], see also [18, Theorem 56], we have continuous operators

$$a(\eta):\mathcal{K}^{s,\gamma}(X^{\wedge})\rightarrow\mathcal{K}^{s-\mu,\gamma-\mu}(X^{\wedge}),\ \ s\in\mathbb{R}.$$

A consequence of $\sigma_0(a) = 0$ is that

$$\tilde{h}(r, w, \tilde{\eta}) \in C^\infty\left(\overline{\mathbb{R}}_+, M_{\mathcal{O}}^{\mu-1}\left(X; \mathbb{R}^q_{\tilde{\eta}}\right)\right). \tag{4.8}$$

This implies

$$\tilde{h}(0, w, \tilde{\eta}) \in M_{\mathcal{O}}^{\mu-1}\left(X; \mathbb{R}^q_{\tilde{\eta}}\right),$$

cf. [18, Remark 39]. We have (3.17) for $h_0(r, w, \eta) = \tilde{h}(0, w, r\tilde{\eta})$, and we write

$$\begin{aligned} a(\eta) = \omega r^{-\mu}\mathrm{Op}_M^{\gamma-n/2}(h - h_0)(\eta)\omega' + \omega r^{-\mu}\mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta)\omega' \\ + m_0(\eta) + g_0(\eta) + m^{\mu-1}(\eta) + g^{\mu-1}(\eta) \end{aligned} \tag{4.9}$$

for

$$m_0(\eta) := r^{-\mu}\omega_\eta \sum_{j=0}^{k} r^j \sum_{|\alpha|=j} \mathrm{Op}_M^{\gamma_{j\alpha}-n/2}(f_{j\alpha})\eta^\alpha \omega'_\eta, \quad g_0(\eta) := \omega\chi(\eta)\sigma_1(g)(\eta)\omega',$$

cf. Definition 2.2 (ii), and

$$m^{\mu-1}(\eta) := (m - m_0)(\eta), \quad g^{\mu-1}(\eta) := (g - g_0)(\eta) \in \mathcal{R}^{\mu-1}_{M+G}(\Omega \times \mathbb{R}^q, \boldsymbol{g}), \tag{4.10}$$

cf. notation (4.1) for $j = 1$. Taylor's formula in the first r-variable in $\tilde{h}(r, w, r\eta) = h(r, w, \eta)$ yields

$$\omega\mathrm{Op}_M^{\gamma-n/2}(h - h_0)(\eta)\omega' = \omega r\mathrm{Op}_M^{\gamma-n/2}(h^{-1})(\eta)\omega' \tag{4.11}$$

for some

$$h^{-1}(r, w, \eta) = \tilde{h}^{-1}(r, w, r\eta), \quad \tilde{h}^{-1}(r, w, \tilde{\eta}) \in C^\infty\left(\overline{\mathbb{R}}_+, M_{\mathcal{O}}^{\mu-1}\left(X; \mathbb{R}^q_{\tilde{\eta}}\right)\right). \tag{4.12}$$

Thus (4.11) belongs to $\mathcal{R}^{\mu-1}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$. It remains to verify that

$$\omega\mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta)\omega' + m_0(\eta) + g_0(\eta) \in \mathcal{R}^{\mu-1}_{\mathrm{M+G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g}).$$

In fact, because of (4.10), (4.11) we have

$$\begin{aligned} \sigma_1(a)(\eta) &= \sigma_1\Big(\omega\mathrm{Op}_M(h_0)\omega' + m_0 + g_0\Big)(\eta) \\ &= \mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta) + \sigma_1(m_0)(\eta) + \sigma_1(g_0)(\eta) \\ &= 0. \end{aligned} \tag{4.13}$$

By virtue of $\sigma_1(a)(\eta) \in L^{\mu}(X^{\Delta}, \boldsymbol{g})_{\text{exit}}$ for every fixed $\eta \neq 0$, cf. Definition 5.3 (iv) below, we can recover the sequence of conormal symbols in a unique way, cf. [23, Subsection 1.3.1, Theorem 4]. Relation (4.13) shows that all conormal symbols vanish. This is the case, in particular, for the leading component, and it follows that

$$\sigma_M^{\mu}\Big(\mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta) + \sigma_1(m_0)(\eta)\Big)(w) = \tilde{h}(0, w, 0) + f_{00}(w) = 0,$$

cf. notation in (3.18), (2.14), and formula (5.12) for $j = 0$. It follows that

$$\sigma_1\Big(\omega \mathrm{Op}_M^{\gamma-n/2}(h_0)\omega' + m_0\Big)(\eta) = 0$$

and hence

$$\omega\Big(\mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta) - \mathrm{Op}_M^{\gamma-n/2}\Big(\tilde{h}\,(0, w, 0)\Big)\Big)\omega' \in \mathcal{R}^{\mu-1}\left(\Omega \times \mathbb{R}^q, \boldsymbol{g}\right).$$

Thus $\omega \mathrm{Op}_M^{\gamma-n/2}(h_0)(\eta)\omega' + m_0(\eta) \in \mathcal{R}^{\mu-1}_{\mathrm{M+G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$, and hence $\sigma_1(g_0)(\eta) = 0$ which entails $g_0(\eta) \in \mathcal{R}^{\mu-1}_{\mathrm{G}}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$. □

Every $a(y, \eta) \in \mathcal{R}^{\mu-j}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma-\mu, (-(k+1), 0])$, $j \in \mathbb{N}$, $j \geq 1$, has again a pair of principal symbols, now of order $\mu - j$, namely,

$$\Big(\sigma_0^{\mu-j}(a), \sigma_1^{\mu-j}(a)\Big) =: \sigma^{\mu-j}(a).$$

For $a(y, \eta) \in \mathcal{R}^{\mu-j}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for any fixed $j \in \mathbb{N}\backslash\{0\}$, we define $\sigma_0^{\mu-j}(a)$ in a similar manner as in Sect. 3 for $j = 0$. In this case we employ Remark 2.1 which gives us the parameter-dependent homogeneous principal symbol $\tilde{p}_{(\mu-j)}(r, x, y, \tilde{\rho}, \xi, \tilde{\eta}) \in S^{(\mu-j)}(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times (\mathbb{R}^{1+n+q}_{\tilde{\rho},\xi,\tilde{\eta}}\backslash\{0\}))$ with $\tilde{p}$ being related with $\tilde{h}$ in (4.4) via Mellin quantisation. Then, similarly as (3.16) we set

$$\sigma_0^{\mu-j}(a)(r, x, y, \rho, \xi, \eta) := r^{-\mu+j}\, \tilde{p}_{(\mu-j)}(r, x, y, r\rho, \xi, r\eta). \tag{4.14}$$

Moreover, $a(y, \eta) \in \mathcal{R}^{\mu-j}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ has a principal edge symbol

$$\sigma_1^{\mu-j}(a)(y, \eta) := r^{-\mu+j} \mathrm{Op}_M^{\gamma-n/2}(h_0)(y, \eta) + \sigma_1^{\mu-j}(m+g)(y, \eta), \tag{4.15}$$

for $h_0(r, y, w, \eta) := \tilde{h}(0, y, w, r\eta)$ and $\sigma_1^{\mu-j}(m+g)(y, \eta)$ given by (4.2).

Theorem 4.2 *For $j \in \mathbb{N}\backslash\{0\}$ the space $\mathcal{R}^{\mu-(j+1)}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma - \mu, (-(k+1), 0])$ is characterised as the set of all $a(y, \eta) \in \mathcal{R}^{\mu}(\Omega \times \mathbb{R}^q, \boldsymbol{g})$ such that*

$$\sigma_0^{\mu-l}(a) = 0, \quad \sigma_1^{\mu-l}(a) = 0$$

for all $l = 0, \ldots, j$.

Proof The result is iterative, and we can apply analogous arguments as for Theorem 4.1. □

Corollary 4.3 *Let M be a manifold with edge Y. Then there is a filtration of $L^{\mu}(M, \boldsymbol{g})$ for $\mu \in \mathbb{R}$, $\boldsymbol{g} = (\gamma, \gamma - \mu, (-(k+1), 0])$, consisting of a sequence of subspaces $L^{\mu-l}(M, \boldsymbol{g})$, $l \in \mathbb{N}$, namely,*

$$L^{\mu}(M, \boldsymbol{g}) \supset L^{\mu-1}(M, \boldsymbol{g}) \supset \cdots \supset L^{\mu-l}(M, \boldsymbol{g}) \supset \cdots \supset L^{-\infty}(M, \boldsymbol{g}), \qquad (4.16)$$

where $L^{\mu-l}(M, \boldsymbol{g}) \subset L^{\mu-l}_{\mathrm{cl}}(M \backslash Y)$ consists of operators A that are represented in an analogous manner as (3.12), here for $A_{\mathrm{int}} \in L^{\mu-l}_{\mathrm{cl}}(M \backslash Y)$, and $a_j(y, \eta) \in \mathcal{R}^{\mu-l}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ for all j. Similar filtrations hold for $L^{\mu}_{\mathrm{M+G}}(M, \boldsymbol{g})$ and $L^{\mu}_{\mathrm{G}}(M, \boldsymbol{g})$, respectively, where $L^{\mu-l}_{\mathrm{M+G}}(M, \boldsymbol{g})$ $(L^{\mu-l}_{\mathrm{G}}(M, \boldsymbol{g}))$ consists of all $A \in L^{\mu-l}(M, \boldsymbol{g})$ such that $A_{\mathrm{int}} = 0$ and $a_j \in \mathcal{R}^{\mu-l}_{\mathrm{M+G}}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g})$ $(\mathcal{R}^{\mu-l}_{G}(\mathbb{R}^q \times \mathbb{R}^q, \boldsymbol{g}))$ for all j.

Note that

$$L^{\mu-l}_{\mathrm{M+G}}(M, \boldsymbol{g}) = L^{\mu-l}_{\mathrm{G}}(M, \boldsymbol{g}) \qquad (4.17)$$

for $l > k$, where $k \in \mathbb{N}$ is determined by the finite weight interval in $\boldsymbol{g}$.

Proposition 4.4 *We have*

$$L^{\mu-l}(M, \boldsymbol{g}) \bigcap L^{-\infty}(M \backslash Y) = L^{\mu-l}_{\mathrm{M+G}}(M, \boldsymbol{g}).$$

Proof Operators $A \in L^{\mu-l}(M, \boldsymbol{g})$ are locally close to the edge in the splitting of variables $(r, x, y) \in \mathbb{R}_+ \times \Sigma \times \Omega$ for open $\Sigma \subseteq \mathbb{R}^n$ and $\Omega \subseteq \mathbb{R}^q$, corresponding to charts on X and Y, respectively, of the form

$$r^{-\mu+l} \mathrm{Op}_M^{\gamma-n/2} \mathrm{Op}_{x,y}(h)$$

cf. notation (5.1) below, for (local) Mellin symbols

$$h(r, x, y, w, \xi, \eta) = \tilde{h}(r, x, y, w, \xi, r\eta)$$

for

$$\tilde{h}\,(r, x, y, w, \xi, \tilde{\eta}) \in S^{\mu-l}_{\mathcal{O}} \left(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}^n_{\xi} \times \mathbb{R}^q_{\tilde{\eta}} \right),$$

where $S^{\mu-l}_{\mathcal{O}}(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}^n_{\xi} \times \mathbb{R}^q_{\tilde{\eta}})$ is the space of all

$$\tilde{h}\,(r, x, y, w, \xi, \tilde{\eta}) \in \mathcal{A}\left(\mathbb{C}_w, S^{\mu-l}_{\mathrm{cl}} \left(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}^n_{\xi} \times \mathbb{R}^q_{\tilde{\eta}} \right) \right)$$

such that

$$\tilde{h}(r, x, y, \beta + i\rho, \xi, \tilde{\eta}) \in S^{\mu-l}_{\mathrm{cl}} \left(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}_{\rho} \times \mathbb{R}^n_{\xi} \times \mathbb{R}^q_{\tilde{\eta}} \right)$$

for all $\beta \in \mathbb{R}$, uniformly in compact β-intervals. If the respective operator belongs to $L^{-\infty}(M\backslash Y)$ then the operator A_{int} is smoothing (A_{int} can be identified with A regarded as an operator $C_0^\infty(M\backslash Y) \to C^\infty(M\backslash Y)$), and

$$h\left(r, x, y, \frac{n+1}{2} - \gamma + i\rho, \xi, \eta\right) \in S^{-\infty}\left(\mathbb{R}_+ \times \Sigma \times \Omega \times \Gamma_{\frac{n+1}{2}-\gamma} \times \mathbb{R}^n_\xi \times \mathbb{R}^q_\eta\right).$$

That means, the homogeneous components $h_{(\mu-l-j)}(r, x, y, \frac{n+1}{2}-\gamma+i\rho, \xi, \eta)$ vanish for $r > 0$ and all j. The symbol $\tilde{h}(r, x, y, \frac{n+1}{2} - \gamma + i\rho, \xi, \tilde{\eta})$ (no matter what the order is) can be reproduced as an asymptotic sum

$$\sum_{j=0}^{\infty} \chi(\rho, \xi, \tilde{\eta})\tilde{h}_{(\mu-l-j)}\left(r, x, y, \frac{n+1}{2} - \gamma + i\rho, \xi, \tilde{\eta}\right)$$

with χ being an excision function in $(\rho, \xi, \tilde{\eta})$, up to an element in $S^{-\infty}(\mathbb{R}_+ \times \Sigma \times \Omega \times \Gamma_{\frac{n+1}{2}-\gamma} \times \mathbb{R}^n_\xi \times \mathbb{R}^q_{\tilde{\eta}})$. This gives us an

$$\tilde{f}\left(r, x, y, \frac{n+1}{2} - \gamma + i\rho, \xi, \tilde{\eta}\right) \in S^{\mu-l}_{\mathrm{cl}}\left(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}_\rho \times \mathbb{R}^n_\xi \times \mathbb{R}^q_{\tilde{\eta}}\right)$$

which is smooth up to $r = 0$. Applying a kernel cut-off operator to $\tilde{f}$ with respect to $w \in \Gamma_{\frac{n+1}{2}-\gamma}$ we recover $\tilde{h} \in S^{\mu-l}_{\mathcal{O}}(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}^n_\xi \times \mathbb{R}^q_{\tilde{\eta}})$ modulo an element of

$$S^{-\infty}\left(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}_\rho \times \mathbb{R}^n_\xi \times \mathbb{R}^q_{\tilde{\eta}}\right).$$

In the present case we have

$$\tilde{h}_{(\mu-l-j)}\left(r, x, y, \frac{n+1}{2} - \gamma + i\rho, \xi, \tilde{\eta}\right) = 0$$

for all $j \in \mathbb{N}$. Thus $\tilde{h}$ itself belongs to $S^{-\infty}_{\mathcal{O}}(\overline{\mathbb{R}}_+ \times \Sigma \times \Omega \times \mathbb{R}^n_\xi \times \mathbb{R}^q_{\tilde{\eta}})$. Now it suffices to note that

$$\omega r^{-\mu+j}\mathrm{Op}_y \mathrm{Op}_M^{\gamma-n/2}\mathrm{Op}_x(h)\omega'$$

is an element of $L^{\mu-l}_{\mathrm{M+G}}(M, \boldsymbol{g})$, more precisely, the operator coming from the local Mellin symbols h for an open covering of X by coordinate neighbourhoods and a sum, using a subordinate partition of unity. □

Remark 4.5 As a byproduct of the proof we obtain

$$h(r, y, w, \eta) = \tilde{h}(r, y, w, r\eta)$$

for an $\tilde{h}(r, y, w, \tilde{\eta}) \in C^\infty(\overline{\mathbb{R}}_+ \times \Omega, M_{\mathcal{O}}^{\mu-l}(X, \mathbb{R}^q_{\tilde{\eta}}))$ such that

$$\tilde{h}\,(r, y, w, \tilde{\eta})\,|_{\mathbb{R}_+ \times \Omega} \in C^\infty\left(\overline{\mathbb{R}}_+ \times \Omega, M_{\mathcal{O}}^{-\infty}\left(X, \mathbb{R}^q_{\tilde{\eta}}\right)\right)$$

automatically belongs to $C^\infty(\overline{\mathbb{R}}_+ \times \Omega, M_{\mathcal{O}}^{-\infty}(X, \mathbb{R}^q_{\tilde{\eta}}))$.

In order to see that edge operator spaces of lower order coincide with the more common definition in terms of vanishing homogeneous principal terms we formulate the pair

$$\sigma^{\mu-l}(A) := \left(\sigma_0^{\mu-l}(A), \sigma_1^{\mu-l}(A)\right)$$

of principal symbols of operators $A \in L^{\mu-l}(M, \boldsymbol{g})$. We define $\sigma_0^{\mu-l}(A)$ as the standard homogeneous principal symbol of A as an element of $L_{\mathrm{cl}}^{\mu-l}(M \backslash Y)$. Locally close to Y we can write

$$\sigma_0^{\mu-l}(A)(r, x, y, \rho, \xi, \eta) = r^{-\mu+l}\tilde{\sigma}_0^{\mu-l}(A)(r, x, y, r\rho, \xi, r\eta) \tag{4.18}$$

for a function $\tilde{\sigma}_0^{\mu-l}(A)(r, x, y, \tilde{\rho}, \xi, \tilde{\eta})$ homogeneous in $(\tilde{\rho}, \xi, \tilde{\eta}) \neq 0$ of order $\mu - l$ and smooth up to $r = 0$. As before for $l = 0$ the symbols occurring in (4.18) when restricted to a neighbourhood close to Y, agree with the symbols involved in (4.14). Clearly $\sigma_0^{\mu-l}$ vanishes on $L_{\mathrm{M+G}}^{\mu-l}(M, \boldsymbol{g})$ since $L_{\mathrm{M+G}}^{\mu-l}(M, \boldsymbol{g}) \subset L^{-\infty}(M \backslash Y)$. Analogously as (3.19) for $A \in L^{\mu-l}(M, \boldsymbol{g})$ we define

$$\sigma_1^{\mu-l}(A)(y, \eta) := \sum_{j=1}^{N} \varphi_j(y)\sigma_1^{\mu-l}(a_j)(y, \eta),$$

using (4.15) for the local edge amplitude functions $a_j \in \mathcal{R}^{\mu-l}(\Omega \times \mathbb{R}^q, \boldsymbol{g}),\ j = 1, \ldots, N$. As a conclusion we recover the filtration property of the edge algebra.

Corollary 4.6 *Let $A \in L^\mu(M, \boldsymbol{g})$ and assume that*

$$\sigma_0^{\mu-l}(A) = 0, \quad \sigma_1^{\mu-l}(A) = 0 \ \text{ for all } \ l = 0, \ldots, j. \tag{4.19}$$

Then we have $A \in L^{\mu-(j+1)}(M, \boldsymbol{g})$.

Thus the property (4.19) for all j yields $A \in \bigcap_{j \in \mathbb{N}} L^{\mu-j}(M, \boldsymbol{g})$. Let us write for the moment

$$L^{-\infty}(\text{symbols}) := \left\{C \in L^\mu(M, \boldsymbol{g}) : \sigma^{\mu-j}(C) = 0 \quad \text{for all } j \in \mathbb{N}\right\}$$

which coincides with $\bigcap_{j \in \mathbb{N}} L^{\mu-j}(M, \boldsymbol{g})$, and

$$L^{-\infty}(\text{mapping}) := \{C \in L^\mu(M, \boldsymbol{g}) : C \text{ has the mapping properties (3.20)}\}.$$

Theorem 4.7 *We have*

$$L^{-\infty}(M, \boldsymbol{g}) = \bigcap_{j \in \mathbb{N}} L^{\mu-j}(M, \boldsymbol{g}) \tag{4.20}$$

for any $\mu \in \mathbb{R}$.

Proof We have to show

$$L^{-\infty}(\text{mapping}) \subseteq L^{-\infty}(\text{symbols}) \tag{4.21}$$

and

$$L^{-\infty}(\text{symbols}) \subseteq L^{-\infty}(\text{mapping}). \tag{4.22}$$

Let us start with (4.21). We consider the case of compact M; the considerations in general only need some simple modifications. The space on the left hand side of (4.20) has been defined by the mapping properties (3.11). Such operators C belong to $L^{-\infty}(M \setminus Y)$ and hence $\sigma_0^{\mu-j}(C)$ vanishes for all $j \in \mathbb{N}$. We have to show that also $\sigma_1^{\mu-j}(C)$ vanishes for all $j \in \mathbb{N}$. To this end we pass to the operator $\varphi\omega C\varphi'\omega'$ for cut-off functions ω, ω' on M, (i.e., $\equiv 1$ in a small neighbourhood of Y, $\equiv 0$ outside another small neighbourhood of Y) and factors $\varphi, \varphi' \in C_0^\infty(G)$ for a coordinate neighbourhood G on Y. We then consider the operator in local coordinates under a chart $\chi|_{V \setminus Y} : V \setminus Y \to X^\wedge \times \mathbb{R}^q$ where V is a wedge neighbourhood such that $V \bigcap Y = G$, cf. formula (3.2) for $\Omega := \mathbb{R}^q$, where $\chi : V \to X^\Delta \times \mathbb{R}^q$ restricts to a chart $G \to \mathbb{R}^q$. Denoting for brevity the operator $\varphi\omega C\varphi'\omega'$ in local coordinates again by C we have to show that the continuity of

$$C : \mathcal{W}^s\left(\mathbb{R}^q, \mathcal{K}^{s,\gamma}\left(X^\wedge\right)\right) \to \mathcal{W}^\infty\left(\mathbb{R}^q, \mathcal{K}_P^{\infty,\gamma-\mu}\left(X^\wedge\right)\right)$$

and

$$C^* : \mathcal{W}^s\left(\mathbb{R}^q, \mathcal{K}^{s,-\gamma+\mu}\left(X^\wedge\right)\right) \to \mathcal{W}^\infty\left(\mathbb{R}^q, \mathcal{K}_Q^{\infty,-\gamma}\left(X^\wedge\right)\right)$$

for all s gives rise to $\sigma_1^{\mu-j}(C) = 0$ for all $j \in \mathbb{N}$. Since this holds for all s it suffices to consider the trivial group action on $\mathcal{W}^s(\mathbb{R}^q, \cdot)$-spaces, i.e., $\kappa = \mathrm{id}$, the action on $\mathcal{W}^\infty(\mathbb{R}^q, \cdot)$ is trivial anyway, cf. notation in Sect. 3. Moreover, it suffices to replace the spaces $\mathcal{K}_P^{\infty,\gamma-\mu}(X^\wedge)$ and $\mathcal{K}_Q^{\infty,-\gamma}(X^\wedge)$ by $E_1 := \mathcal{K}_\Delta^{\infty,\gamma-\mu}(X^\wedge)$ and $E_2 := \mathcal{K}_\Delta^{\infty,\gamma}(X^\wedge)$ for a weight interval $\Delta = (-\delta, 0]$ for some sufficiently small $\delta > 0$, cf. (5.3) and Remark 5.2 below. The spaces E_1, E_2 are nuclear and Fréchet. It is then easy to recognise that C has a kernel in

$$c(y, y') \in C^\infty\left(\mathbb{R}^q \times \mathbb{R}^q, E_1 \hat{\otimes}_\pi H_1\right) \bigcap C^\infty\left(\mathbb{R}^q \times \mathbb{R}^q, H_2 \hat{\otimes}_\pi E_2\right) \tag{4.23}$$

for $H_1 = \mathcal{K}^{-s,-\gamma}(X^\wedge)$, $\quad H_2 = \mathcal{K}^{s,-\gamma+\mu}(X^\wedge)$. Then, similarly as in scalar smoothing operators, the integral operator

$$Cu(y) = \int c(y, y')u(y')dy'$$

can be written in the form

$$Cu(y) = \iint e^{i(y-y')\eta} c(y, y')\psi(\eta)u(y')e^{-i(y-y')\eta} dy' đ\eta$$

for a $\psi \in C_0^\infty(\mathbb{R}^q)$ such that $\int \psi(\eta)đ\eta = 1$. Thus C has a double symbol

$$a(y, y', \eta) = c(y, y')\psi(\eta)e^{-i(y-y')\eta}$$

which is a Schwartz function in $\eta \in \mathbb{R}^q$ with values in $E_1 \hat{\otimes}_\pi H_1 \bigcap H_2 \hat{\otimes}_\pi E_2$ and with smooth dependence on $(y, y') \in \mathbb{R}^q \times \mathbb{R}^q$. By construction C is also properly supported with respect to (y, y')-variables. We can pass to a left symbol $a_{\mathrm{L}}(y, \eta) \sim \sum_{\alpha \in \mathbb{R}^q} \frac{1}{\alpha!} D_{y'}^\alpha \partial_y^\alpha a(y, y', \eta)|_{y=y'}$. Since $\sigma_1^{\mu-j}(C)$ only depends on the summands for $|\alpha| \leq j$ which are all Schwartz functions in η it follows that $\sigma_1^{\mu-j}(C)(y, \eta) = 0$ for all j.
The second part of the proof consists of verifying (4.22). In other words we prove the continuities (3.11) for any $C \in \bigcap_{j \in \mathbb{N}} L^{\mu-j}(M, \boldsymbol{g})$. The idea of proving (3.13) for $A \in L^{\mu-j}(M, \boldsymbol{g})$ for all j is analogous to that for $j = 0$. In the present case, for $C \in \bigcap_{j \in \mathbb{N}} L^{\mu-j}(M, \boldsymbol{g})$ we already know that $\sigma_0^{\mu-j}(C) = 0$ for all j, i.e., $C \in L^{-\infty}(M \backslash Y)$. Thus we have $C \in L^{\mu-j}(M, \boldsymbol{g}) \bigcap L^{-\infty}(M \backslash Y)$ and hence $C \in L_{\mathrm{M+G}}^{\mu-j}(M, \boldsymbol{g})$. For sufficiently large j we even have $C \in L_{\mathrm{G}}^{\mu-j}(M, \boldsymbol{g})$, cf. relation (4.17). This gives us the continuity of

$$C : H^{s,\gamma}(M) \to H_P^{s-\mu+j,\gamma-\mu}(M)$$

for every sufficiently large j, for some asymptotic type P depending on C. Since this holds for all j, it follows that C has the desired mapping property in (3.11). For C^* we can argue in an analogous manner, using that formal adjoints of operators in the edge calculus with weight data $\boldsymbol{g}$ belong the calculus with weight data $\boldsymbol{g}^* := (-\gamma + \mu, -\gamma, (-(k+1), 0])$. This completes the proof of Theorem 4.7. □

5 Asymptotics and operators on cones

The analysis of edge operators refers to spaces with discrete asymptotics and pseudo-differential operators on infinite cones. Here we give some notation.

Throughout this exposition we write

$$\mathrm{Op}_x(a)u(x) = \iint e^{i(x-x')\xi} a(x, x', \xi)u(x')dx' đ\xi \tag{5.1}$$

for pseudo-differential operators in $\mathbb{R}^n \ni x$ (or open subsets of $\mathbb{R}^n$) with double symbols $a(x, x', \xi)$ of some order, $đ\xi = (2\pi)^{-n} d\xi$.

By a discrete Mellin asymptotic type R we understand a sequence

$$R = \{(r_j, n_j)\}_{j \in \mathbb{J}} \subset \mathbb{C} \times \mathbb{N} \tag{5.2}$$

for some index set $\mathbb{J} \subseteq \mathbb{Z}$ such that $\Pi_{\mathbb{C}} R := \{r_j\}_{j \in \mathbb{J}}$ intersects the strip $\{c \leq \operatorname{Re} w \leq c'\}$ in a finite set for every $c \leq c'$.

A function $\chi \in C^\infty(\mathbb{C})$ is called an R-excision function if $\chi(w) = 0$ for $\operatorname{dist}(w, \Pi_{\mathbb{C}} R) < \varepsilon_0,\ \chi(w) = 1$ for $\operatorname{dist}(w, \Pi_{\mathbb{C}} R) > \varepsilon_1$, for some $0 < \varepsilon_0 < \varepsilon_1$.

Let X be a smooth closed manifold of dimension n, and let R be a Mellin asymptotic type.

Definition 5.1 By $M_R^{-\infty}(X)$ we denote the space of all

$$f \in \mathcal{A}\left(\mathbb{C} \setminus \Pi_{\mathbb{C}} R, L^{-\infty}(X)\right)$$

which are meromorphic with poles at all r_j of multiplicity $n_j + 1$, and finite rank Laurent coefficients at $(w - r_j)^{-(k+1)}$, $0 \leq k \leq n_j$, and $\chi f|_{\Gamma_\beta} \in L^{-\infty}(X; \Gamma_\beta)$ for any R-excision function χ and every $\beta \in \mathbb{R}$, uniformly in compact β-intervals.

The space $M_R^{-\infty}(X)$ is a Fréchet space in a natural way. Set

$$M_R^\mu(X) := M_{\mathcal{O}}^\mu(X) + M_R^{-\infty}(X)$$

in the Fréchet topology of the non-direct sum.

Let us now pass to spaces of distributions with asymptotics of type P. Considering a discrete asymptotic type

$$P = \left\{(p_j, m_j)\right\}_{j=0,\ldots,N} \subset \mathbb{C} \times \mathbb{N},$$

$N \in \mathbb{N} \cup \{\infty\}$, we say that P is associated with the weight data (γ, Θ), for a weight interval $\Theta = (\vartheta, 0],\ \ -\infty \leq \vartheta < 0$, if

$$\Pi_{\mathbb{C}} P := \{p_j\}_{j=0,\ldots,N} \subset \left\{ \frac{n+1}{2} - \gamma + \vartheta < \operatorname{Re} w < \frac{n+1}{2} - \gamma \right\},$$

$n = \dim X$, and N is finite for $\vartheta > -\infty$, while $\operatorname{Re} p_j \to -\infty$ as $j \to \infty$ for $N = \infty$ and $\vartheta = -\infty$.

For convenience we assume everywhere that P satisfies the shadow condition, i.e., $(p, m) \in P$ entails $(p - k, m) \in P$ for every $k \in \mathbb{N}$ with

$$\operatorname{Re} p - k > \frac{m+1}{2} - \gamma + \vartheta.$$

An example of such a P is $T = \{(j,0)\}_{j\in\mathbb{N}}$, the Taylor asymptotic type. Let us set

$$\mathcal{K}_{\Theta}^{s,\gamma}(X^{\wedge}) := \varprojlim_{\varepsilon>0} \mathcal{K}^{s,\gamma-\vartheta-\varepsilon}(X^{\wedge}). \tag{5.3}$$

Let P be associated with (γ, Θ). For a fixed cut-off function ω and finite Θ we form the space

$$\mathcal{E}_P(X^{\wedge}) := \left\{ \omega(r)\sum_{j=0}^{N} r^{-p_j}\sum_{k=0}^{m_j} c_{jk}(x)\log^k r : c_{jk}\in C^{\infty}(X) \right\},$$

where $0 \le k \le m_j$, $j = 0,\dots,N$. We then have $\mathcal{E}_P(X^{\wedge}) \bigcap \mathcal{K}_{\Theta}^{s,\gamma}(X^{\wedge}) = \{0\}$, and we set

$$\mathcal{K}_P^{s,\gamma}(X^{\wedge}) := \mathcal{K}_{\Theta}^{s,\gamma}(X^{\wedge}) + \mathcal{E}_P(X^{\wedge}), \tag{5.4}$$

which is a direct sum. The space $\mathcal{K}_P^{s,\gamma}(X^{\wedge})$ for any asymptotic type P and finite Θ is a Fréchet space with group action κ, see (2.12). In fact it can be written as a projective limit

$$\mathcal{K}_P^{s,\gamma}(X^{\wedge}) = \varprojlim_{j\in\mathbb{N}} E^j \tag{5.5}$$

of Hilbert spaces for $E^0 := \mathcal{K}^{s,\gamma}(X^{\wedge})$ and continuous embeddings $E^j \hookrightarrow E^0$ for all j, where E^0 is endowed with $\kappa = \{\kappa_\delta\}_{\delta\in\mathbb{R}_+}$ like (2.12) which restricts to a group action on E^j for every $j \in \mathbb{N}$.
For $\Theta = (-\infty, 0]$ we form $\Theta_k := (-(k+1), 0]$, $P_k := \{(p,m) \in P : \operatorname{Re} p > \frac{n+1}{2} - (k+1)\}$. Then we have the spaces $\mathcal{K}_{P_k}^{s,\gamma}(X^{\wedge})$ for every $k \in \mathbb{N}$, and define

$$\mathcal{K}_P^{s,\gamma}(X^{\wedge}) := \varprojlim_{k\in\mathbb{N}} \mathcal{K}_{P_k}^{s,\gamma}(X^{\wedge}). \tag{5.6}$$

Similarly as (2.13) we set

$$\mathcal{K}_P^{s,\gamma;e}(X^{\wedge}) := [r]^{-e}\mathcal{K}_P^{s,\gamma}(X^{\wedge}), \quad \mathcal{K}_P^{\infty,\gamma;\infty}(X^{\wedge}) := \bigcap_{s,e\in\mathbb{R}} \mathcal{K}_P^{s,\gamma;e}(X^{\wedge}),$$

for every $s, \gamma, e \in \mathbb{R}$; also these spaces are endowed with the group action κ.

Remark 5.2 For any asymptotic type P associated with the weight data (γ, Θ) there is a $\delta > 0$ such that for $\Delta := (-\delta, 0]$ we have continuous embeddings

$$\mathcal{K}_P^{s,\gamma;e}(X^{\wedge}) \hookrightarrow \mathcal{K}_{\Delta}^{s,\gamma;e}(X^{\wedge}).$$

In fact, it suffices to set $\delta = \operatorname{dist}(\Pi_{\mathbb{C}}P, \Gamma_{\frac{n+1}{2}-\gamma})$.

Edge symbols take values in the cone algebra over the infinite cone X^{Δ}, cf. (2.1). So we briefly recall what we understand by the cone algebra. In cone pseudo-differential operators on an infinite cone X^{Δ} we start observing the behaviour for $r \to \infty$, the conical exit to infinity. In this case the variables $(r, x) \in \mathbb{R}_+ \times X$ are considered for x in a coordinate neighbourhood U on X that we identify with $x \in \mathbb{R}^n$. Then, a standard process via an open covering of X and a subordinate partition of unity gives us classical operators globally on $\mathbb{R}_+ \times X$ for $r \to \infty$, indicated by $L_{\mathrm{cl}}^{\mu;\nu}(\cdot)_{\mathrm{exit}}$ for a pair of orders $(\mu; \nu) \in \mathbb{R} \times \mathbb{R}$. The local definition is as follows. Consider the space

$$S^{\mu;\nu}\left(\mathbb{R}^{n+1}_{\tilde{x}} \times \mathbb{R}^{n+1}_{\tilde{\xi}}\right) \subset C^\infty\left(\mathbb{R}^{n+1}_{\tilde{x}} \times \mathbb{R}^{n+1}_{\tilde{\xi}}\right)$$

defined by symbolic estimates

$$\left|D^\alpha_{\tilde{x}} D^\beta_{\tilde{\xi}} a(\tilde{x}, \tilde{\xi})\right| \le c_{\alpha\beta} \langle\tilde{\xi}\rangle^{\mu-|\beta|} \langle\tilde{x}\rangle^{\nu-|\alpha|}$$

for all $\alpha, \beta \in \mathbb{N}^{n+1}$ and $(\tilde{x}, \tilde{\xi}) \in \mathbb{R}^{n+1} \times \mathbb{R}^{n+1}$, for constants $c_{\alpha\beta} > 0$. The space $L^{\mu;\nu}(\mathbb{R}^{n+1})_{\mathrm{exit}}$ is defined as the set of all operators $\mathrm{Op}_{\tilde{x}}(a)$ for arbitrary $a(\tilde{x}, \tilde{\xi}) \in S^{\mu;\nu}(\mathbb{R}^{n+1}_{\tilde{x}} \times \mathbb{R}^{n+1}_{\tilde{\xi}})$. The subspace of classical operators is defined in terms of symbols in $S^\mu_{\mathrm{cl}}(\mathbb{R}^{n+1}_{\tilde{\xi}}) \hat{\otimes}_\pi S^\nu_{\mathrm{cl}}(\mathbb{R}^{n+1}_{\tilde{x}})$. The corresponding space with classical symbols is denoted by

$$L^{\mu;\nu}_{\mathrm{cl}}\left(\mathbb{R}^{n+1}\right)_{\mathrm{exit}}. \tag{5.7}$$

This notation has an extension to $\mathbb{R}_+ \times X$ for a smooth manifold X, which gives us the spaces

$$L^{\mu;\nu}(\mathbb{R}_+ \times X)_{\mathrm{exit}} \quad \text{or} \quad L^{\mu;\nu}_{\mathrm{cl}}(\mathbb{R}_+ \times X)_{\mathrm{exit}}.$$

More details can be found in [24, Subsection 1.4].

Definition 5.3 Let X be a closed smooth manifold.

(i) The space $L_{\mathrm{G}}(X^\Delta, \boldsymbol{g})$ of Green operators on X^Δ for $\mu \in \mathbb{R}$, $\boldsymbol{g} = (\gamma, \gamma - \mu, \Theta)$, is the set of all operators

$$G \in \bigcap_{s,e\in\mathbb{R}} \mathcal{L}\left(\mathcal{K}^{s,\gamma;e}\left(X^\wedge\right), \mathcal{K}_P^{\infty,\gamma-\mu;\infty}\left(X^\wedge\right)\right)$$

such that

$$G^* \in \bigcap_{s,e\in\mathbb{R}} \mathcal{L}\left(\mathcal{K}^{s,-\gamma+\mu;e}\left(X^\wedge\right), \mathcal{K}_Q^{\infty,-\gamma;\infty}\left(X^\wedge\right)\right)$$

for some G-dependent asymptotic types P and Q.

(ii) By $L_{\mathrm{M+G}}(X^\Delta, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma - \mu, (-(k+1), 0])$, $k \in \mathbb{N}$, we define the space of all $M + G$ for $G \in L_{\mathrm{G}}(X^\Delta, \boldsymbol{g})$ and smoothing Mellin operators

$$M := r^{-\mu}\omega \sum_{j=0}^{k} r^j \mathrm{Op}_M^{\gamma_j - n/2}(f_j)\omega' \tag{5.8}$$

for cut-off functions $\omega,\ \omega'$, and smoothing Mellin symbols $f_j(w) \in M^{-\infty}_{R_j}(X)$ with Mellin asymptotic types R_j, and weights $\gamma_j \in \mathbb{R}$, satisfying the conditions

$$\gamma - j \leq \gamma_j \leq \gamma,\ \Pi_{\mathbb{C}} R_j \cap \Gamma_{\frac{n+1}{2}-\gamma_j} = \varnothing.$$

For $\boldsymbol{g} = (\gamma, \gamma-\mu, (-\infty, 0])$ we define $L_{\mathrm{M+G}}(X^{\Delta}, \boldsymbol{g})$ as the intersection of the respective $L_{\mathrm{M+G}}$ spaces for $(\gamma, \gamma-\mu, (-(k+1), 0])$ over $k \in \mathbb{N}$.
(iii) The space $L^{\mu}(X^{\Delta}, \boldsymbol{g})$ for $\boldsymbol{g} = (\gamma, \gamma-\mu, \Theta),\ \ \mu \in \mathbb{R},\ \ \Theta = (-(k+1), 0], k \in N \cup \{\infty\}$, is defined as the set of all operators

$$A = r^{-\mu}\big\{\omega \mathrm{Op}_M^{\gamma-n/2}(h)\omega' + (1-\omega)\mathrm{Op}_r(p)(1-\omega'')\big\} + M + C \qquad (5.9)$$

for cut-off functions $\omega'' \prec \omega \prec \omega'$, arbitrary $h(r, w) \in C^{\infty}(\overline{\mathbb{R}}_+, M^{\mu}_{\mathcal{O}}(X))$ and $p(r, \rho)$ given by

$$p(r,\rho) := \tilde{p}(r, r\rho),\ \ \tilde{p}(r,\tilde{\rho}) \in C^{\infty}\left(\overline{\mathbb{R}}_+, L^{\mu}_{\mathrm{cl}}\left(X; \mathbb{R}_{\tilde{\rho}}\right)\right),$$

M as in (ii) and $C \in L^{-\infty}(X^{\Delta}, \boldsymbol{g})$, i.e.,

$$C : H^{s,\gamma}_{\mathrm{comp}}(X^{\Delta}) \to H^{\infty,\gamma-\mu}_{\mathrm{loc},P}(X^{\Delta}),\ \ C^* : H^{s,-\gamma+\mu}_{\mathrm{comp}}(X^{\Delta}) \to H^{\infty,-\gamma}_{\mathrm{loc},Q}(X^{\Delta})$$

for every $s \in \mathbb{R}$ and asymptotic types P and Q, depending on C.

(iv) The space $L^{\mu}(X^{\Delta}, \boldsymbol{g})_{\mathrm{exit}}$ for $\boldsymbol{g} = (\gamma, \gamma-\mu, \Theta),\ \mu \in \mathbb{R}, \Theta$ as in (iii), is defined as the set of all operators

$$A = A_{\psi} + M + G \qquad (5.10)$$

for

$$A_{\psi} = r^{-\mu}\big\{\omega \mathrm{Op}_M^{\gamma-n/2}(h)\omega' + \varphi \mathrm{Op}_r(p)\varphi' + \chi P_{\mathrm{exit}}\chi'\big\} \qquad (5.11)$$

where h, p are as in (iii), $\ \omega \prec \omega'$ are arbitrary cut-off functions, $\varphi \prec \varphi' \in C_0^{\infty}(\mathbb{R}_+)$, and $\chi \prec \chi'$ excision functions (*i.e.*, $1-\chi \succ 1-\chi'$ are cut-off functions in the former sense), where $\omega + \varphi + \chi = 1,\ \ M + G \in L_{\mathrm{M+G}}(X^{\Delta}, \boldsymbol{g})$ and

$$P_{\mathrm{exit}} \in L^{\mu;0}_{\mathrm{cl}}(\mathbb{R}_+ \times X)_{\mathrm{exit}}.$$

Clearly we have

$$L^{\mu}(X^{\Delta}, \boldsymbol{g})_{\mathrm{exit}} \subset L^{\mu}(X^{\Delta}, \boldsymbol{g}).$$

Let us finally recall the notion of conormal symbols of operators in $L^{\mu}(X^{\Delta}, \boldsymbol{g})$. By that we understand the operator functions

$$\sigma_M^{\mu-j}(A)(w) = \frac{1}{j!}\left(\partial_r^j h\right)(0, w) + f_j(w), \qquad (5.12)$$

$j = 0, \ldots, k$, with $k \in \mathbb{N}$ being involved in $\boldsymbol{g}$. For $j = 0$ we also write $\sigma_M(A) := \sigma_M^\mu(A)$, called the principal conormal symbol.

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